

A weak gravity conjecture for scalar field theories

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ABSTRACT: We put forward a conjecture for a class of scalar field theories analogous to the recently proposed weak gravity conjecture [4] for U(1) gauge field theories. Taking gravity into account, we find an upper bound on the gravity interaction strength, expressed in terms of scalar coupling parameters. This conjecture is supported by some two-dimensional models and noncommutative field theories.

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Scalars, such as Higgs and inflaton, play significant roles in both particle physics and cosmology. Recently, Arkani-Hamed et.al. conjectured an upper bound on the strength of gravity relative to gauge forces in quantum gravity [4]. Their conjecture enriches the criteria of consistent effective field theory proposed in [2, 3]. These criteria help to constrain the string landscape [1] in the vast vacua of string theory. After a careful investigation, we find that a similar bound exists in a class of scalar field theories. In this class of theories at least, “gravity is the weakest” even in the appearance of scalars.

We will study scalar field theories with soliton solutions. we will focus our attention on a special class of such theories. In these theories, the coefficient of the mass term is $\pm\frac{1}{2}\mu^2$ with $\mu^2 > 0$, and there are higher order terms controlled by a coupling constant λ in addition to μ . So the theories are described by only two parameters: the mass parameter μ and the coupling constant λ . When the scalar is coupled to gravity, we find an interesting constraint on μ and λ .

The idea is to study a scalar system coupled to two-dimensional dilaton gravity, which is assumed to be dimensional reduction of some consistent quantum gravity system in higher dimensions. In two dimensions, the system contains solitons in the weak gravity limit (plus some other massive particles in a sector not considered here). These solitons appear as asymptotic scattering states so there is a well-defined S-matrix. As one increases the gravity coupling, these solitons disappear, thus there are no more massive particles and the S-matrix ceases to exist. We assume that in this case the quantum system becomes inconsistent, because without the S-matrix, only correlation functions exist which are not observables in a gravity theory. We therefore conjecture that the quantum theory exists only in the weak gravity region. In fact, in some exact S-matrices, such as the Sine-Gordon model [11, 12] and the chiral field model [13], only solitons appear in spectra. It is a matter subject to debate that whether this is also true in higher dimensions, but we feel that a consistent theory in higher dimensions should result in a consistent theory upon dimensional reduction, thus we lift this conjecture to higher dimensions, in particular, to four dimensions.

When the scalar is coupled to two-dimensional dilaton gravity, the action is generally taken to be

$$S = \frac{1}{2\pi} \int d^2x \sqrt{-\gamma} e^{-2\phi} \left[\frac{1}{8G} (R + 4(\nabla\phi)^2) - \frac{1}{2}(\nabla\varphi)^2 \mp \frac{1}{2}\mu^2\varphi^2 - V_c(\varphi, \lambda, \mu) \right] \quad (1)$$

In this notation, the potential of scalar φ is $V = V_c \pm \frac{1}{2}\mu^2\varphi^2$. In most cases, after a redefinition of λ , the higher order coupling terms V_c can be rewritten as

$$V_c(\varphi, \lambda, \mu) = \frac{\mu^2}{g^{2\alpha}} V'_c(g^\alpha\varphi) \quad (2)$$

in which $g = \frac{\sqrt{\lambda}}{\mu}$, and the value of α depends on the form of V_c . As we will show later, for a given V_c , the value of α can be worked out explicitly.

We rescale the coordinates and the scalar to make them dimensionless

$$t' = \mu t, \quad x' = \mu x, \quad \varphi' = g^\alpha\varphi \quad (3)$$

the action (1) becomes

$$S = \frac{1}{2\pi} \int d^2x' \sqrt{-\gamma} e^{-2\phi} \left[\frac{1}{8G} (R' + 4(\nabla'\phi)^2) + \frac{1}{g^{2\alpha}} \left(-\frac{1}{2}(\nabla'\varphi')^2 \mp \frac{1}{2}\varphi'^2 - V'_c(\varphi') \right) \right]. \quad (4)$$

We stress that rescaling λ in (2) by a numeric factor will rescale the expression of V'_c also by a numeric factor. To remove this uncertainty, when writing (2), one should define λ appropriately to get a reasonable V'_c whose numeric coefficients are of order 1.

From (4), it is clear that the gravitational strength relative to the strength of scalar interaction is controlled by $\frac{G}{g^{2\alpha}}$. If $g^{2\alpha} \gg G$, the effect of gravity is just a small correction to the original scalar field theory, so one can treat the weak gravity perturbatively, expanding in terms of $\frac{\sqrt{G}}{g^\alpha}$. On the other hand, if $g^{2\alpha} \ll G$, the correction would be large enough to destroy the original solutions and make the theory inconsistent. Hence quantum gravity provides a criterion of consistent scalar field theories, parametrically

$$g^{2\alpha} \gtrsim G \quad (5)$$

This constraint can be lifted to higher dimensions:

$$\left(\frac{\lambda}{\mu^2} \right)^\alpha \gtrsim G \quad (6)$$

In D -dimensional spacetime, the Newton constant G has the dimensions of $mass^{2-D}$ while the coupling constant λ has the dimensions of $mass^{2-\frac{D-2}{\alpha}}$.

In the special case

$$V = -\frac{1}{2}\mu^2\varphi^2 + \frac{1}{n}\lambda\varphi^n + \left(\frac{1}{2} - \frac{1}{n} \right) \mu^2 \left(\frac{\mu^2}{\lambda} \right)^{\frac{2}{n-2}} \quad (7)$$

with an even number $n > 2$, we have $\alpha = \frac{2}{n-2}$, therefore the relation (6) becomes

$$\left(\frac{\lambda}{\mu^2} \right)^{\frac{2}{n-2}} \gtrsim G \quad (8)$$

In four dimensions, setting $n = 4$ leads to the constraint $\frac{\mu}{\sqrt{\lambda}} \lesssim M_{Pl}$.

In the examples studied below, when gravity decouples, solitons are in the spectrum. Disappearance solitons in the strong gravity singals breakdown of unitarity: when we tune up gravity interaction strength gradually, the rank of S-matrix will have a jump at some point in the parameter space. Indeed, we do not even know if the S-matrix exists in the strong gravity region. So for the parameter region violating (6), there is no consistent quantum gravity theory. We suspect that the same kind of relation (6) remains true in a larger class of theories without solitons, although we do not have evidence.

A kink is the most familiar soliton in two-dimensional spacetime. The action of the coupled system reads

$$S = \frac{1}{2\pi} \int d^2x \sqrt{-\gamma} e^{-2\phi} \left[R + 4(\nabla\phi)^2 - \frac{1}{2}(\nabla\varphi)^2 - V \right] \quad (9)$$

the equations of motion are

$$\begin{aligned}
 0 &= R + 4\nabla^2\phi - 4(\nabla\phi)^2 - \frac{1}{2}(\nabla\varphi)^2 - V \\
 0 &= 2\gamma^{ab}\nabla_a\varphi\nabla_b\phi - \nabla^2\varphi + \frac{dV}{d\varphi} \\
 0 &= -\frac{1}{2}\nabla_a\varphi\nabla_b\varphi + 2\nabla_a\nabla_b\phi + \gamma_{ab}\left[2(\nabla\phi)^2 - 2\nabla^2\phi + \frac{1}{4}(\nabla\varphi)^2 + \frac{1}{2}V\right]
 \end{aligned} \tag{10}$$

with $a, b = 0, 1$ in two dimensions.

We are interested in static solutions, in which the dilaton ϕ , the scalar field φ and the metric γ_{ab} are independent of t .

For any two-dimensional static metric

$$\begin{aligned}
 ds^2 &= \gamma_{00}dt^2 + 2\gamma_{01}dtdx + \gamma_{11}dx^2 \\
 &= -\gamma_{00}\left[-\left(dt + \frac{\gamma_{01}}{\gamma_{00}}dx\right)^2 + \frac{\gamma_{01}^2 - \gamma_{00}\gamma_{11}}{\gamma_{00}^2}dx^2\right]
 \end{aligned} \tag{11}$$

By redefining coordinates

$$d\tilde{t} = dt + \frac{\gamma_{01}}{\gamma_{00}}dx, \quad d\tilde{x} = \sqrt{\frac{\gamma_{01}^2 - \gamma_{00}\gamma_{11}}{\gamma_{00}^2}}dx \tag{12}$$

one can always switch to the conformal gauge $\tilde{\gamma}_{ab} = e^{2w}\eta_{ab}$ while keeping the solutions static. Henceforth, we will work in conformal gauge and omit the tilde for brevity.

The equations of motion (10) now reduce to

$$2\left(\frac{d\phi}{dx}\right)^2 - 2\frac{d^2\phi}{dx^2} + \frac{1}{4}\left(\frac{d\varphi}{dx}\right)^2 + \frac{1}{2}e^{2w}V = -\frac{d^2w}{dx^2} \tag{13}$$

$$2\left(\frac{d\phi}{dx}\right)^2 - 2\frac{d^2\phi}{dx^2} + \frac{1}{4}\left(\frac{d\varphi}{dx}\right)^2 + \frac{1}{2}e^{2w}V = -2\frac{dw}{dx}\frac{d\phi}{dx} \tag{14}$$

$$2\left(\frac{d\phi}{dx}\right)^2 - 2\frac{d^2\phi}{dx^2} + \frac{1}{4}\left(\frac{d\varphi}{dx}\right)^2 + \frac{1}{2}e^{2w}V = 2\frac{dw}{dx}\frac{d\phi}{dx} + \frac{1}{2}\left(\frac{d\varphi}{dx}\right)^2 - 2\frac{d^2\phi}{dx^2} \tag{15}$$

$$2\frac{d\varphi}{dx}\frac{d\phi}{dx} - \frac{d^2\varphi}{dx^2} + e^{2w}\frac{dV}{d\varphi} = 0 \tag{16}$$

We have written them in forms that the left hand sides of (13), (14) and (15) are the same. Equating the right hand sides of (13) and (14), we find a simple relation

$$\frac{dw}{dx} = Ce^{2\phi} \tag{17}$$

Combining it with (14) and (15), it follows that

$$4Ce^{2\phi}\frac{d\phi}{dx} = 2\frac{d^2\phi}{dx^2} - \frac{1}{2}\left(\frac{d\varphi}{dx}\right)^2 \tag{18}$$

Using (14) and (15) to eliminate $\frac{dw}{dx}$, we obtain

$$4\left(\frac{d\phi}{dx}\right)^2 - 2\frac{d^2\phi}{dx^2} + e^{2w}V = 0 \quad (19)$$

Combined with (16) to eliminate e^{2w} , it gives

$$2\frac{dV}{d\varphi}\left[\frac{d^2\phi}{dx^2} - 2\left(\frac{d\phi}{dx}\right)^2\right] = V\left(\frac{d^2\varphi}{dx^2} - 2\frac{d\varphi}{dx}\frac{d\phi}{dx}\right) \quad (20)$$

We will restrict our attention to a kink potential $V(\varphi)$ with $\alpha = 1$ in (2). Based on the work of 't Hooft [6], we assume

$$\varphi = \frac{1}{g}f_{-1}(x) + \text{subleading terms} \quad (21)$$

where we have used the notation of $g = \frac{\sqrt{\lambda}}{\mu}$, while $f_{-1}(x)$ is some function independent of g .

Remember that in the notation of (4), the potential and its derivative with respect to φ can be written in the form

$$\begin{aligned} V(\varphi, \lambda, \mu) &= \frac{\mu^2}{g^2}V'(g\varphi) = \frac{\mu^2}{g^2}V'(\varphi') \\ \frac{dV}{d\varphi} &= \frac{\mu^2}{g^2}\frac{dV'}{d\varphi'}\frac{d\varphi'}{d\varphi} = \frac{\mu^2}{g}\frac{dV'}{d\varphi'} \end{aligned} \quad (22)$$

In the leading order $\varphi' \sim f_{-1}$, therefore

$$\begin{aligned} V &= \frac{\mu^2}{g^2}V'(f_{-1}) + \text{subleading terms} \\ \frac{dV}{d\varphi} &= \frac{\mu^2}{g}\frac{dV'}{df_{-1}} + \text{subleading terms} \end{aligned} \quad (23)$$

Substitute the leading order terms of (21) and (23) into (20), we find

$$2\frac{dV'}{df_{-1}}\left[\frac{d^2\phi}{dx^2} - 2\left(\frac{d\phi}{dx}\right)^2\right] = \frac{1}{g^2}V'(f_{-1})\left(\frac{d^2f_{-1}}{dx^2} - 2\frac{df_{-1}}{dx}\frac{d\phi}{dx}\right) \quad (24)$$

In order to be consistent with this equation, ϕ has to take the form

$$\phi = \frac{1}{g^2}\Phi_{-2} + \text{subleading terms} \quad (25)$$

In the strong gravity limit $g \ll 1$, the value of ϕ in (25) will generate a divergent value of w in (17), unless $C = 0$. So we set $C = 0$ from now on in order to avoid the essential singularity.

We shall use the perturbative expansion to study equations (18) and (20) with $C = 0$ in the weak gravity limit and the strong gravity limit respectively.

In the weak gravity region, we assume $g \gg 1$ and expand ϕ and φ in $\frac{1}{g}$

$$\begin{aligned} \varphi &= \frac{1}{g}f_{-1} + \frac{1}{g^2}f_{-2} + \dots \\ \phi &= \frac{1}{g^2}\Phi_{-2} + \frac{1}{g^3}\Phi_{-3} + \dots \end{aligned} \quad (26)$$

Substituting (26) into (18) and (20), we have the following lowest order equations

$$\begin{aligned} \frac{d^2\Phi_{-2}}{dx^2} &= \frac{1}{4} \left(\frac{df_{-1}}{dx} \right)^2 \\ \frac{d^2\Phi_{-2}}{dx^2} &= \frac{1}{2} \frac{V}{\frac{dV}{df_{-1}}} \frac{d^2 f_{-1}}{dx^2} \end{aligned} \tag{27}$$

So we have

$$\frac{V}{\frac{dV}{df_{-1}}} \frac{d^2 f_{-1}}{dx^2} = \frac{1}{2} \left(\frac{df_{-1}}{dx} \right)^2 \tag{28}$$

It is the same as the kink equation in [6] after a simple calculation

$$\frac{d\varphi}{dx} = \sqrt{2V} \tag{29}$$

We conclude that in weak gravity region the kink solution survives when this scalar field is coupled to gravity.

The equation of motion of the scalar field (28) reduce to the equations of motion for scalar field theory in flat spacetime. So if there are 2-soliton solutions in the pure scalar field theory, there should be when the scalar field theory is coupled to gravity. On the other hand, from (27) we see that even there is no soliton, there may be linear dilaton, and it does no harm to the one soliton solution. With the soliton, the second derivative of the dilaton becomes positive in some localized region, and zero elsewhere. So asymptotically the soliton will induce another linear dilaton. But the effect is only a shift of the coefficient of the linear dilaton in pure gravity, and hence it will do harm to two soliton solution. Thus, if S matrix exists in the original pure scalar field theory, it will exist when the scalar field theory is coupled to dilaton gravity.

In the strong gravity region, we take the limit $g \ll 1$ and expand ϕ and φ in g

$$\begin{aligned} \varphi &= \frac{1}{g} f_{-1} + f_0 + \dots \\ \phi &= \frac{1}{g^2} \Phi_{-2} + \frac{1}{g} \Phi_{-1} + \dots \end{aligned} \tag{30}$$

Plugging (30) into (18) and (20) leads to the following lowest order equations

$$\begin{aligned} \frac{d^2\Phi_{-2}}{dx^2} &= \frac{1}{4} \left(\frac{df_{-1}}{dx} \right)^2 \\ \frac{d\Phi_{-2}}{dx} &= \frac{1}{2} \frac{V}{\frac{dV}{df_{-1}}} \frac{df_{-1}}{dx} \end{aligned} \tag{31}$$

We will analyze the equations in two particular models, i.e. the φ^4 model and the Sine-Gordon model.

1. $V(\varphi) = \frac{\lambda}{4} \left(\varphi^2 - \frac{\mu^2}{\lambda} \right)^2$

Substituting this potential into the equations (31), we have the solution

$$\begin{aligned} f_{-1} &= e^{-8C_1x+C_2} \\ \Phi_{-2} &= \frac{1}{16} e^{-16C_1x+2C_2} + C_1x + C_3 \end{aligned} \tag{32}$$

From (32), it can be seen easily that f_{-1} does not satisfy the kink condition, so the kink solution breaks down in this case.

2. $V(\varphi) = \frac{\mu^4}{\lambda} \left[1 - \cos\left(\frac{\sqrt{\lambda}}{\mu}\varphi\right) \right]$

Inserting this potential into the equations (31), we have the solution

$$\begin{aligned} f_{-1} &= 2 \arctan \sinh(D_1 x + D_2) \\ \Phi_{-2} &= \ln \cosh(D_1 x + D_2) + D_3 \end{aligned} \tag{33}$$

From the potential in this case, we find that this solution connects two maximum points rather than minimum, so there is no kink solution in this case.

In action (9) we have set $8G = 1$. By recovering G , one finds that the expansion parameter is not g but $\frac{g}{\sqrt{G}}$. This accounts for the name “weak/strong gravity”.

The calculation above tells us that kink solutions appear in weak gravity limit $\frac{g}{\sqrt{G}} \gg 1$ but disappear in strong gravity limit $\frac{g}{\sqrt{G}} \ll 1$. In the absence of gravity, the existence of soliton is dictated by unitarity. Naturally we expect that unitarity will still require solitons when gravity is switched on. Therefore, when $\frac{g}{\sqrt{G}} \ll 1$ the breakdown of kink solutions implies that we cannot take the strong gravity limit smoothly. In other words, there must be a lower bound on the value of $\frac{g}{\sqrt{G}}$, which is determined by the critical point of the equations of motion. Parametrically, the critical value is of order 1. This leads to

$$\left(\frac{\lambda}{\mu^2}\right) \gtrsim G \tag{34}$$

which agrees with the conjecture (6). It would be useful to work out the exact value of $\frac{g}{\sqrt{G}}$ at the critical point.

All the discussions above are on the classical level. One may wonder whether the conclusion is reliable on the quantum level. To check this, we should consider some quantum corrections such as higher derivative corrections. When we consider the original action with the correction term $(\alpha'R)^n$, then the equations of motion (13), (14) and (15) will deserve the correspondent correction terms, proportional to some positive power of R . Note that our discussion below (12) shows that we can always take the conformal gauge and set the field static in 2 dimensional spacetime. So we will always have $R = -2e^{-2\omega} \frac{d^2\omega}{dx^2}$. So fortunately these terms will vanish under the classical solution (17) with $C = 0$, e.g, $\frac{d\omega}{dx} = 0$. So we can conclude that our results must be preserved at the quantum level.

Another piece of evidence comes from noncommutative solitons. In noncommutative field theories, when the noncommutativity parameter θ is sufficiently large (but finite), there are stable soliton solutions in some three-dimensional scalar theories [7, 8]. Based on a cubic potential, in [9], Huang and She found a relation involving the noncommutative parameter. In the following, we will briefly repeat part of their work.

Consider a scalar field theory in three-dimensional noncommutative spacetime, whose Euclidean action is

$$S = \int d^3x \sqrt{-\gamma} \left(\frac{1}{2} \gamma^{ij} \partial_i \varphi \partial_j \varphi + V(\varphi) \right). \tag{35}$$

Written in terms of the canonically commuting noncommutative coordinates

$$z = \frac{x^1 + ix^2}{\sqrt{\theta}}, \bar{z} = \frac{x^1 - ix^2}{\sqrt{\theta}}, \quad (36)$$

the energy is

$$E = \int d^2z \left(\frac{1}{2}(\partial\varphi)^2 + \theta V(\varphi) \right). \quad (37)$$

When θV is large, the potential energy dominates and we can find an approximate solitonic solution by solving the equation $\frac{dV}{d\varphi} = 0$.

In our notations, the scalar potential is $V(\varphi) = \frac{1}{2}\mu^2\varphi^2 + \frac{1}{3}\lambda\varphi^3$. The value of the potential at the stationary point $\frac{dV}{d\varphi} = 0$, namely $\varphi_{cr} = -\frac{\mu^2}{\lambda}$, determines the soliton energy completely [8]

$$E = 2\pi\theta V(\varphi_{cr}) = \frac{\pi}{3} \frac{\theta\mu^6}{\lambda^2} \quad (38)$$

In three-dimensional spacetime, a massive pointlike soliton will generate a deficit angle in the metric. The requirement that the deficit angle be less than 2π implies

$$8\pi GE < 2\pi \quad (39)$$

Observing (38) and (39), we obtain

$$\frac{1}{G} \gtrsim \frac{\theta\mu^6}{\lambda^2} \quad (40)$$

Actually, for a general potential (not necessarily of polynomial form, see [7]) with a noncommutative soliton whose energy is positive

$$V = \frac{\mu^2}{g^{2\alpha}} V'_c(g^\alpha\varphi) \pm \frac{1}{2}\mu^2\varphi^2 = \frac{\mu^2}{\left(\frac{\lambda}{\mu^2}\right)^\alpha} \left[V'_c(\varphi') \pm \frac{1}{2}\varphi'^2 \right] \quad (41)$$

there is a similar relation

$$\frac{1}{G} \gtrsim E \sim \frac{\theta\mu^2}{\left(\frac{\lambda}{\mu^2}\right)^\alpha} \quad (42)$$

On the other hand, the condition for the existence of soliton solution in the noncommutative theory is

$$\theta \geq \frac{1}{\mu^2}. \quad (43)$$

Combine (42) and (43), a constraint with the same form of (6) can be obtained. Using (43) on the right hand side of (42), we get

$$\frac{1}{G} \gtrsim \frac{\theta\mu^2}{\left(\frac{\lambda}{\mu^2}\right)^\alpha} \geq \frac{1}{\left(\frac{\lambda}{\mu^2}\right)^\alpha}, \quad (44)$$

which is just the the form of (6). This is a hint that our conjecture (6) is universal.

One may notice that in U(1) gauge field theories the weak gravity conjecture has a sharp form [4]: there are always particles with $M \lesssim Q$. However, it is difficult to similarly sharpen our conjecture (6). The particle charge Q is naturally defined for U(1) gauge

field theories but not for scalar field theories. Fortunately, it has been shown that in [4] their conjecture can be expressed in different forms, one of which is $T_{el} \lesssim \frac{g}{\sqrt{G}}$. This form is similar to conjecture (6) for scalar field theories. Essentially, in both U(1) gauge field theories and scalar field theories, the weak gravity conjecture is a statement about effective field theories. So it can be used to identify the swampland, a series of effective field theories which are consistent semiclassically yet inconsistent in a quantum gravity theory.

We have so far been dealing with real scalar field theories. The generalization to theories of complex scalars and scalar multiplets is not straightforward. However, we have some hints. In [5], we have offered some evidence supporting the weak gravity conjecture of gauge field theories. To avoid possible confusion, let us use e to replace g in [5]. By noticing $m_W^2 = 2e^2 \frac{\mu^2}{\lambda}$, one can easily obtain the relation (6) for three-dimensional examples studied in [5].

In [10], the weak gravity conjecture in [4] has been generalized from flat spacetime to dS/AdS spacetime, leading a bound on cosmological constant. It is also interesting to study scalar theory in a dS background.

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